

Probing near extremal black holes with D-branes

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Abstract

We calculate the one loop effective action for D-brane probes moving in the presence of near BPS D-branes. The v^2 term agrees with supergravity in all cases and the static force agrees for a five dimensional black hole with two large charges. It also agrees qualitatively in all the other cases. We make some comments on the M(atric) theory interpretation of these results.

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1. Introduction

D-branes are localized probes of the spacetime geometry [1][2][3][4]. When a D-brane probe is in the presence of other D-branes there are massive open strings stretching between the probe and the other branes. When these massive open strings are integrated out one obtains an effective action for the massless fields representing the motion of the probe. This can be interpreted as the action of a D-brane moving on a nontrivial supergravity background. In many cases one can find the exact supergravity backgrounds in this fashion [1][2][4]. Most of the backgrounds analyzed previously correspond to BPS supergravity solutions. The calculation in [5] of D-brane probes moving in a nonsingular extremal black hole background shows agreement for the one loop term but the status of the two loop term is not clear.

In this paper we study probes moving in near BPS backgrounds. In the D-brane language this corresponds to a D-brane probe moving close to other D-branes which are not in their ground state but that have some additional (small) energy above extremality.

We calculate the one loop effective action for a variety of near extremal configurations and we compare it to the corresponding supergravity results. Since the background is no longer BPS and supersymmetry is broken there is no reason for the forces to cancel. Indeed there is a net force on a static probe. We also calculate the v^2 force and we find agreement with supergravity in *all* cases. We compute the static force for a D-brane configuration with Q_5 D-fivebranes carrying also Q_1 D-onebrane charge and some extra energy. We find precise agreement. The static force agrees qualitatively in all the other cases. All one loop calculations of this type reduce to evaluating F^4 terms in the gauge theory.

2. The one loop calculation

In this section we will consider a D-brane probe in the presence of some other D-branes. By integrating out the stretched open strings we calculate the one-loop effective action for the probe. More concretely, we consider $N + 1$ D-branes, N of which sit at the origin ($r = 0$) and the last is the probe which sits at a distance r . At low energies and for small separations the system is described by a $p + 1$ dimensional $U(N + 1)$ Yang-Mills theory with 16 supersymmetries broken down to $U(N) \times U(1)$ by the expectation value of an adjoint scalar which measures the distance from the probe to the rest of the branes. The fields with one index in $U(N)$ and the other on $U(1)$ are massive, with a mass $m = r/(2\pi\alpha')$. If we integrate them out we get an effective action for the light degrees of

freedom. All D-branes have $p + 1$ worldvolume dimensions, with $p \leq 6$. The action of the Yang Mills theory is

$$S_0 = -\frac{1}{g(2\pi)^p} \int d^{1+p}x \frac{1}{4} \text{Tr}[F_{\mu\nu}F^{\mu\nu}] + \text{fermions} , \quad (2.1)$$

where μ, ν are ten dimensional indices [6]. We normalize the string coupling g so that $g \rightarrow 1/g$ under S-duality and set $\alpha' = 1^2$. The gauge field is a hermitian matrix and the trace is defined without any additional coefficients.

We will calculate the leading order terms in the field strength F . We will assume that F is slowly varying so that we can neglect derivatives, DF , as well as commutators, $[F, F]$. Commutators are small if covariant derivatives of the fields are small since $[D, D]F \sim [F, F]$. A similar approach was taken in [9] to propose a form for the non-abelian generalization of the Dirac-Born-Infeld action.

We consider first the case where the probe D-brane is static. The one loop contribution to the effective action will come from a one loop diagram with four external lines corresponding to light $U(N)$ fields on the excited D-branes. Diagrams with less than four external lines cancel due to supersymmetry.

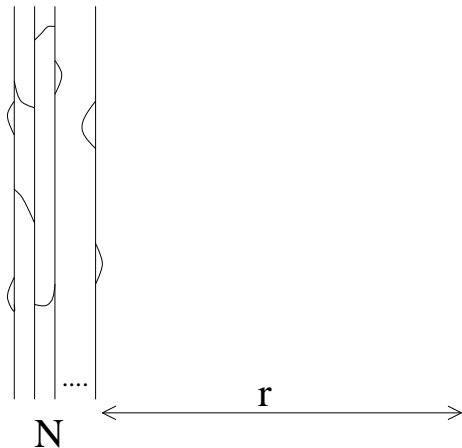


FIGURE 1: D-brane configuration. N D-branes are sitting together carrying low energy excitations and the probe is separated by a distance r .

Instead of doing the field theory calculation we do the string theory calculation. It corresponds to a one loop string diagram with the string ending on the probe on one side

² These conventions are the ones used in in [7][8].

and the excited brane on the other. In order to find the term that we are interested in, it is enough to do the calculation for constant field strength F . Since we are going to neglect commutators we can take it to be diagonal in the group indices. This calculation was done in [10][11][12][13] and it is a very simple generalization of the calculations done by [14][15][16][4][17]. We refer to those papers for the details.

The result is

$$S_1 = \frac{c_{7-p}}{8(2\pi)^p r^{7-p}} \int d^{1+p}x \left\{ \text{Tr}[F_{\mu\nu} F^{\nu\rho} F_{\rho\sigma} F^{\sigma\mu}] - \frac{1}{4} \text{Tr}[F_{\mu\nu} F^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}] \right\} , \quad (2.2)$$

where μ, ν are again ten dimensional indices and c_q is the numerical constant

$$c_q = (4\pi)^{\frac{q}{2}-1} \Gamma(\frac{q}{2}) . \quad (2.3)$$

We see that there are two terms (remember that we are neglecting commutators, $[F, F] \sim 0$) with a structure that is independent of the dimensionality of the brane. This structure is the same as the structure of the F^4 terms in the Dirac-Born-Infeld action [9][13]. We have done the calculation for diagonal field strengths but we wrote (2.2) in terms of the full non-abelian F , which is consistent with our approximations. The result (2.2) is correct to all orders in α' for reasons explained in [4], so the gauge theory calculation, which was a priori correct only for $r \ll \sqrt{\alpha'}$, also gives the right result for $r \gg \sqrt{\alpha'}$, a fact that is important in M(atrrix) theory [18].

The case of branes moving with constant velocity corresponds to $F_{0i} = v^i$ and we get [14][15][4]

$$S = \frac{c_{7-p}}{8r^{7-p}} R^p \int dt (v^2)^2 , \quad (2.4)$$

where R is the radius of each of the p compact directions. This is the right form for the v^4 term in supergravity.

We also see that if F describes a BPS excitation (2.2) vanishes. The two possible BPS excitations are traveling waves (momentum along the brane) and instantons (for $p \geq 4$) describing a $p-4$ brane inside a p brane [19]. For a traveling wave we only have $F_{-i} \neq 0$ where $x^- = t - x^9$, and both terms in (2.2) individually cancel. For the case of the instanton let us denote by I, J, K, L the dimensions along which the gauge field is nontrivial (the four dimensions of the instanton). Using the self duality condition we find

$$F_{IJ} F^{JK} = \frac{1}{4} \delta_{IK} F_{LJ} F^{JL} , \quad (2.5)$$

so that the two terms in (2.2) cancel. We also obtain a cancelation if we have instantons and traveling waves at the same time.

2.1. Calculation of the static force for the five dimensional black hole

Now we want to excite the N branes and evaluate this term in some generic thermal ensemble. This is difficult in principle because we have not calculated the fermionic terms and they will contribute. We will do the calculation in a case where it is easy to see what the effect of the fermions is. Of course, as explained in [20] the supergravity solution is expected to agree only in the limit of large gN . Which means that the effective large N coupling of the gauge theory is strong.

We will calculate (2.2) in a configuration carrying the charges of the the five dimensional near extremal black hole of [21][22] in the dilute gas regime. Then $p = 5$, we call $N = Q_5$ and we also put Q_1 instanton strings along one of the directions of the fivebrane, let us call it the direction $\hat{9}$. Even though they are called “instantons” these objects are physically string solitons of the 1+5 dimensional gauge theory. The instanton configuration is characterized by some moduli ξ^r , $r = 1, \dots, 4Q_1Q_5$. These moduli can oscillate when we move along the direction $\hat{9}$, $\xi(t, x_9)$. We are interested in the case where the energy of the oscillations is small, so that we can describe the excitations of the system as oscillations in moduli space. The condition is that the total energy due to the oscillations should be much smaller than the energy of the instantons $E \ll R_9Q_1/g$. (In the notation of [7][23] this means $r_n \ll r_1$.) This picture of the D-1-brane charge being carried by instantons in the gauge theory is correct when the energy of the instantons is much smaller than the total mass of the fivebranes $M_1 = R_9Q_1/g \ll M_5 = R_5R_6R_7R_8R_9Q_5/g$ (this means $r_1 \ll r_5$). So that we are in the dilute gas regime of [23]³ Calling $x^\pm = x^9 \pm t$, the nonzero components of the gauge field are $F_{\pm I} = \partial_\pm \xi^r \partial_r A_I$ with $I = 5, 6, 7, 8$ and F_{IJ} , the field of the instanton. The action for the small fluctuations of the instanton configuration becomes

$$\begin{aligned} S_0 &= \frac{1}{g(2\pi)^5} \int d^{1+5}x \frac{1}{2} \text{Tr}[F_{\alpha I} F^{\alpha I}] + \text{fermions} \\ &= \frac{1}{2} \int dt dx^9 G_{rs}(\xi) \partial_\alpha \xi^r \partial^\alpha \xi^s + \text{fermions} , \end{aligned} \tag{2.6}$$

where $\alpha = \pm$. We have an nonlinear sigma model action for the instanton fluctuations [20]. The theory (2.6) has (4,4) supersymmetry and the metric G_{rs} is hyperkähler [20].

Using (2.5) we can see that the effective action (2.2) reduces to

$$S_1 = \frac{2}{(2\pi)^5 r^2} \int d^{1+5}x \text{Tr}[F_{+I} F_{+I} F_{-J} F_{-J}] + \text{fermions} . \tag{2.7}$$

³ The definition of the dilute gas regime $r_n \ll r_1, r_5$ does not require any specific relation between r_1 and r_5 .

When we integrate over t, x^9 we will effectively average separately the term with $++$ and the ones proportional to $--$. We assume that the oscillations average the fields in such a way that we can replace $F_{+I}F_{+I}$ by its average value, both in spacetime indices and group indices. So we get

$$\begin{aligned}
S_1 &= \frac{2}{(2\pi)^5 r^2} \int dt \int d^5x \text{Tr}[F_{+I}F_{+I}] \times \frac{1}{(2\pi)^5 RV Q_5} \int d^5x \text{Tr}[F_{-J}F_{-J}] \\
&= \frac{2g^2}{RV Q_5} \frac{1}{r^2} \int dt \int dx^9 T_{++} \int dx^9 T_{--} \\
&= \frac{2g^2 E_L E_R}{RV Q_5} \frac{1}{r^2} \int dt
\end{aligned} \tag{2.8}$$

where R is the radius of the 9^{th} direction and $V = R_5 R_6 R_7 R_8$ is the product of the radii in the other four directions. $E_{L,R}$ are the left and right moving energies of the effective two dimensional theory (2.6). Notice that we have calculated only the bosonic terms. Supersymmetry then implies that the fermions appear in (2.8) just as another energy contribution. The form of the operator in (2.8) is the identical to the one that appeared in the calculation of the fixed scalars greybody factors [24].

2.2. Calculation of the v^2 forces

If the probe is also moving, it will feel a force proportional to v^2 besides the static force. This is an effect which, in some sense, is of the same order of magnitude as the static force. The static force is roughly equal to the square of the brane excitation energy, while the v^2 force will be proportional to the energy, we are free to vary the ratio of the energy above extremality and the velocity.

Now we go back to the general case of $N + 1$ parallel p branes. In order to calculate the v^2 force we have to compute a one loop diagram with two insertions corresponding to probe fields and two insertions corresponding to the fields of the other N D-branes. We can calculate a general term involving the gauge field F_1 on the $U(N)$ piece and the gauge field F_2 on the $U(1)$ part. We do the general calculation because it is very simple. Again, we think in terms of string theory. We take the gauge fields to be diagonal in group indices but not necessarily commuting in the spacetime indices. If we think in terms of the open string stretching between the two D-branes we see that the boundary conditions will be

$$\begin{aligned}
\partial_+ X^\mu(\sigma = 0) &= \left(\frac{1 - F_1}{1 + F_1} \right)^\mu_{\nu} \partial_- X^\nu(\sigma = 0) , \\
\partial_+ X^\mu(\sigma = 2\pi) &= \left(\frac{1 - F_2}{1 + F_2} \right)^\mu_{\nu} \partial_- X^\nu(\sigma = 2\pi) ,
\end{aligned} \tag{2.9}$$

where F is regarded as a matrix with respect to the spacetime indices [11].

We see that this is equivalent to the boundary conditions with

$$F_1 \rightarrow \tilde{F}, \quad F_2 \rightarrow 0, \quad (2.10)$$

where

$$\left(\frac{1 - \tilde{F}}{1 + \tilde{F}} \right)^\mu_{\nu} = \left(\frac{1 - F_1}{1 + F_1} \right)^\mu_{\delta} \left(\frac{1 + F_2}{1 - F_2} \right)^\delta_{\nu}. \quad (2.11)$$

In the presence of F_1 and F_2 the effective action will be as in (2.2) but in terms of \tilde{F} as defined by (2.11). Since all the lower order terms in \tilde{F} vanish we only need to calculate \tilde{F} to first order in the field strength which gives $\tilde{F} = F_1 - F_2$. Replacing this in (2.2) we find

$$\begin{aligned} S_2 = & \frac{c_{7-p}}{8(2\pi)^p r^{7-p}} \int d^{1+p}x \, 4\text{Tr}[F_{1\mu\nu} F_1^{\nu\rho}] \text{Tr}[F_{2\rho\sigma} F_2^{\sigma\mu}] + 2\text{Tr}[F_{1\mu\nu} F_{1\rho\sigma}] \text{Tr}[F_2^{\nu\rho} F_2^{\sigma\mu}] \\ & - \frac{1}{2} \text{Tr}[F_{1\mu\nu} F_1^{\mu\nu}] \text{Tr}[F_{2\rho\sigma} F_2^{\rho\sigma}] - \text{Tr}[F_{1\mu\nu} F_{1\rho\sigma}] \text{Tr}[F_2^{\mu\nu} F_2^{\rho\sigma}] \\ & + \text{fermions}. \end{aligned} \quad (2.12)$$

We have written a separate trace over the group indices of the probe to generalize the result to the case of $U(N+M) \rightarrow U(N) \times U(M)$ but in the discussion below we consider just $M=1$.

We will be giving the probe some velocity $F_{0i} = v^i$. In this case we see that (2.12) reduces to

$$S_2 = \frac{c_{7-p}}{r^{7-p}} \int d^{1+p}x \, \left[\frac{v^2}{2} g T_{00} - \frac{1}{2(2\pi)^p} \text{Tr}[F_{1i\mu} F_{1j}^{\mu}] v^i v^j + \text{fermions} \right], \quad (2.13)$$

where

$$T_{00} = \frac{1}{g(2\pi)^p} \left\{ \text{Tr}[F_{1\mu 0} F_1^{\mu}{}_{0}] + \frac{1}{4} \text{Tr}[F_{1\mu\nu} F_1^{\mu\nu}] + \text{fermions} \right\} \quad (2.14)$$

is the stress tensor associated to the unbroken $U(N)$ Yang-Mills theory. Supersymmetry implies that the fermions will appear in (2.14) contributing to the stress tensor.

We are interested in the case where the velocity is along the directions transverse to the branes. We mean that the probe Wilson lines do not have a constant time dependence ($F_{2I0} = 0$), in other words, there are no winding fundamental strings dissolved on the probe. We are also interested in evaluating (2.13) in an average sense. Under this

conditions the term proportional to $v^i v^j$ in (2.13) does not contribute since it will be proportional to $D_\mu X^i D^\mu X^j$, we can integrate by parts the covariant derivative and then the term vanishes using the equations of motion. This implies that (2.13) becomes

$$S_2 = \frac{c_{7-p} g}{r^{7-p}} E \int dt \frac{v^2}{2} , \quad (2.15)$$

where E is the total energy on the D-brane. This result is valid for all branes in all dimensions. It includes as a particular case the one loop calculation of [5]. In that case, $p = 5$ and we have an extremal state containing Q_1 instanton strings and momentum N . The total energy on the fivebrane worldvolume is [7]

$$E = \frac{N}{R} + \frac{RQ_1}{g} , \quad (2.16)$$

so that the probe metric becomes

$$S = \frac{RV}{g} \int dt \frac{v^2}{2} \left(1 + \frac{g^2 N}{R^2 V r^2} + \frac{gQ_1}{V r^2} + o(g^2/r^4) \right) , \quad (2.17)$$

where the $o(g^2/r^4)$ term should come from a two loop calculation [5].

3. Supergravity calculation

We start by writing the action of a test D-brane in the background of a near extremal black hole that carries D-brane charge [25]

$$S = -\frac{1}{g(2\pi)^p} \int d^{p+1}x e^{-\phi} \sqrt{\det G} + A_{01\dots p} , \quad (3.1)$$

where G is the induced metric on the brane and A_{p+1} is the $p+1$ form potential that couples to the D-brane charge. The background is the corresponding supergravity solution [26]. For a solution carrying a single charge we find

$$S = -\frac{R^p}{g} \int dt \left[f^{-1} \sqrt{h - f(\dot{r}^2/h + r^2 \dot{\Omega}^2)} + \frac{\sinh 2\alpha}{2} \frac{r_0^{7-p}}{r^{7-p}} f^{-1} \right] , \quad (3.2)$$

where

$$f = 1 + \frac{r_0^{7-p} \sinh^2 \alpha}{r^{7-p}} , \quad h = 1 - \frac{r_0^{7-p}}{r^{7-p}} , \quad (3.3)$$

where r_0 and α are parameters related to the D-brane charge, N , and the mass above extremality, E ,

$$c_{7-p}N = \frac{r_0^{7-p} \sinh 2\alpha}{2g}, \quad E = \frac{(9-p)R^p r_0^{7-p}}{2(7-p)c_{7-p}g^2}, \quad (3.4)$$

with c_{7-p} as in (2.3). Expanding (3.2) in powers of the velocity we find a static potential, a v^2 term, etc. We start calculating the v^2 term

$$\frac{R^p}{g} \int dt \frac{1}{2} \frac{1}{\sqrt{h}} \left(\frac{\dot{r}^2}{h} + r^2 \dot{\Omega}^2 \right) \quad (3.5)$$

We notice that the coefficient is different for \dot{r}^2 and $r^2 \dot{\Omega}^2$ while in our result (2.15) the coefficient was the same. The resolution to this apparent contradiction is that the coordinate r of the spacetime solution is not necessarily the same as the coordinate r of the D-brane calculation. Let us define a new radial coordinate ρ by the equation

$$\frac{d\rho}{\rho} = \frac{dr}{r\sqrt{h}}. \quad (3.6)$$

Using (3.6) the term (3.5) becomes proportional to $v_\rho^2 = (\dot{\rho}^2 + \rho^2 \dot{\Omega}^2)$, so that ρ is interpreted as the distance that appears in the D-brane calculation.⁴ Expanding (3.2) to leading order in r_0^{7-p}/r^{7-p} , taking into account the change of coordinates (3.6) we get the velocity dependent term

$$S = \frac{R^p}{g} \int dt \frac{v_\rho^2}{2} \left(1 + \frac{(9-p)}{2(7-p)} \frac{r_0^{7-p}}{\rho^{7-p}} \right). \quad (3.7)$$

Expressing r_0^{7-p} in terms of the energy (3.4) we find again (2.15), in precise agreement with the D-brane probe calculation. Notice that it was crucial to perform a change of variables to find agreement. This quantity agrees for any p , this is due to the simplicity of the operator that couples to v^2 : it is just the physical energy, so it is not renormalized by strong coupling effects (large gN effects).

Now let us turn to the static potential

$$V = \frac{R^p}{g} f^{-1} \left[\sqrt{h} - 1 + \left(\frac{\sinh 2\alpha}{2} - \sinh^2 \alpha \right) \frac{r_0^{7-p}}{r^{7-p}} \right]. \quad (3.8)$$

⁴ Note that the horizon at $r = r_0$ is not at $\rho = 0$ as it said in a previous version of this paper, I thank E. Kiritsis for pointing that out.

To leading order in r_0^{7-p}/r^{7-p} and leading order in $c_{7-p}gN/r^{7-p}$, (which means large distances) we find

$$V = -\frac{R^p}{g} \frac{r_0^{7-p}}{2e^{2\alpha}r^{7-p}} \quad (3.9)$$

Using (3.4) we see that (3.9) is proportional to E^2/N which roughly agrees with (2.2).

We can also calculate the static potential for the case of a fivebrane probe on a five dimensional near extremal black hole background carrying Q_5 D-fivebrane charge and Q_1 D-onebrane charge. We obtain the same formula for the static potential as in (3.9) (for $p = 5$) but the formulas expressing the energies of the left and right movers in terms of the parameter r_0 are different [7] and lead to [5]

$$S = -\frac{2g^2 E_L E_R}{RVQ_5} \frac{1}{r^2} \quad (3.10)$$

in precise agreement with the the D-brane calculation (2.8).

4. M(atr)ix theory black holes

According to the proposal of Banks, Fishler, Shenker and Susskind [18], in order to calculate any process in M-theory we should first boost the system along some direction so that it carries a large longitudinal momentum. The D-brane theory of black holes can be naturally interpreted in terms of M(atr)ix theory [27][28].

The most natural black holes to consider are one dimensional extended solutions along the longitudinal dimension which also carry longitudinal momentum. They are homogeneous, translational invariant (but not boost invariant) along the longitudinal direction and localized in the transverse directions. An example is the extremal black hole solution corresponding to zero branes lifted up to 11 dimensions. This solution is singular and should not be taken too seriously very close to the horizon. We could, however, consider a near extremal solution which indeed has a well defined horizon and is nonsingular. Alternatively we could compactify 5 or more dimensions and get extremal black holes with nonzero horizon area by turning on some other charges. As we boost along the longitudinal dimension the black hole becomes closer to extremality. In the finite N proposal of Susskind [29], we can also have these momentum carrying black holes since they are supergravity solutions which are left invariant by translations along the null direction, so they are also solutions to M-theory compactified along a null direction.

We see that if R is the radius of the compactified direction we will have to take the limit

$$\frac{r_g^{d-3}}{l_p^{d-3}} \sim N_h \frac{l_p^2}{R^2} \rightarrow \infty \quad (4.1)$$

in order to trust the black hole supergravity solution. We define r_g to be the typical gravitational radius of the configuration, as the radius of the d-2 transverse sphere were the redshift with the asymptotic observer becomes of order one (i.e. the metric becomes different by an order one amount from the Minkowski metric). N_h is the total momentum carried by the black hole which will be $N_h \leq N$ than the total momentum of the system.

In [18][30][31][16] it was shown that the gravitons indeed feel the right forces when they are in the presence of some other graviton, or a two brane, etc. If one had a system of zero branes (some gravitons) which is excited by some finite energy above extremality we expect that the system will stay for some time in the “stadium” region of [3][4] where the noncommuting properties of the matrices becomes important. In the number of zero branes becomes large as in (4.1) then we expect that the system will become a near extremal black hole.

A graviton propagating close to this near BPS configuration will feel a force. The static force is proportional to the square of the energy of the excited system of gravitons in the light cone frame, this energy is the same as the energy above extremality defined above. If we have M theory on T^p we have to consider a $p + 1$ dimensional YM theory on the dual torus [18][32], more precisely, the corresponding theory coming from a nontrivial fixed point. Since this calculation is IR dominated and finite in the gauge theory, we expect that the answer is independent of the precise UV definition of the theory. For general p there is agreement up to a numerical coefficient for the leading power in $1/r$ of the force. In the case of M(atric) theory on T^5 and a configuration with Q_1 longitudinal fivebranes the agreement is precise. This configuration is a black string in the six uncompactified dimensions. The description [28][27] is almost the same as the description of the 5 dimensional black hole with Q_5 D-fivebranes and Q_1 D-fivebranes. A difference is that in matrix theory we only have the gauge theory on the branes, we do not have the infinite tower of massive states. The gauge theory calculation, which in string theory is valid only for $r \ll \sqrt{\alpha'}$, will now have to be valid also for distances $r \gg \sqrt{\alpha'}$.

The agreement is more impressive for the v^2 force. This force is universally proportional to the energy. This calculation can be viewed as a test of the equivalence principle in M(atric) theory.

We have concentrated in the one loop contribution, it would be interesting to study further corrections due to perturbative and non-perturbative effects [33].

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